QUASI–GEODESIC FLOWS

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This is a short description of Erasmus+ lectures at Universidad de Granada, Spain to give on April 16–20, 2018. For participants, no special knowledge on flows (or more generally, foliations) is assumed. Basic topological and geometrical tools will be provided but some experience in hyperbolic geometry is useful.

After Danny Calegari and Steven Frankel we describe a structure of quasi–geodesic flows on 3–dimensional hyperbolic manifolds.

A flow in an action of the addirtive group $\mathbb{R}$ o on given manifold. We concentrate on closed 3–dimensional hyperbolic manifolds i.e. having locally isometric covering by the hyperbolic space $\mathbb{H}^3$. Such flow is quasi–geodesic if every flowline of the lifted flow is a quasi–geodesic in $\mathbb{H}^3$. Quasi–geodesic flows are probably the only reasonable metric objects which are foliations of hyperbolic 3–manifolds which do not carry neither geodesic foliation in any dimension (Zeghib) nor quasi–geodesic foliations of dimension 2 (Fenley).

We start with foundations of hyperbolic manifolds, hyperbolic groups and their asympotic properties. Then we describe shortly notions for foliations and flows mentioning their type like (quasi)–isometric, (quasi)–geodesic, (pseudo)–Anosov etc.

We take care of topology of the plane focusing on decompositions into continua. For such decompositions we construct a circular order in the set of their topological ends.

Since after Calegari any quasi–geodesic flow on a hyperbolic 3 manifold has the Hausdorff flowspace (i.e. the plane) we are able to apply decompositions for a compactification the flowspace by ends of flowlines making it a closed disc.

Finally, we study new results of Frankel on extension properties of quasi–geodesic flow. In particular we take a look for its proof of Calegari conjecture stating that such flows need to have closed orbits.

At the end, we add some remarks about (quasi)–geodesic flows in non-compact case.

References

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[4] S. Frankel, *Coarse hyperbolicity and closed orbits for quasigeodesic flows*, arXiv:1507.04320v2.

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