COMPLEX HYPERBOLIC GEOMETRY

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This Erasmus+ serie of lectures is planned for November 12–16, 2018 at Universidad de Granada, Spain. No special preparation is needed, the standard facts from (real) hyperbolic geometry will be recalled.

The n–dimensional complex hyperbolic space $\mathbb{C}H^n$ consists of classes of negative vectors with respect to Hermitian form of signature $(n,1)$ in $\mathbb{C}^{n+1}$. There are two useful models in $\mathbb{C}^n$: the ball model and the paraboloid model which are closely related to the ball and half–space models in the real case. In particular considerations, we restrict to one of the models and observe exapmles in (complex) dimension 2.

$\mathbb{C}H^n$ with the Hermitian form is an Hadamard manifold of strictly negative sectional curvature ranging from $-\frac{1}{4}$ (real directions) to -1 (complex directions) so it has visible ideal boundary. In the paraboloid model the ideal boundary has a structure of Heisenberg group.

In $\mathbb{C}H^n$ there are two types of totally geodesic submanifolds. One of them, a *complex hyperbolic subspace*, comes from intersection of $\mathbb{C}H^n$ by a complex linear subspace and is isometric to $\mathbb{C}H^k$ for $k < n$. The others are *totally real hyperbolic subspaces* coming from totally real linear subspaces. They are embedding of rescaled $\mathbb{H}^k$'s. On the ideal boundary we observe *chains* being boundaries of complex hyperbolic subspaces.

In $\mathbb{C}H^n$ no totally geodesic hypersurface (of real codimension 1) exists although bisectors (i.e. hypersurfaces equidistant from psir of points) have interesting properties.

The isometry group $PU(n,1)$ of $\mathbb{C}H^n$ consists of classes of matrices preserving the Hermitian form. We study them with connection to their boundary action.

As a final, we give distance formulae for geodesic, complex geodesics and hyperspaces. For it, we develop some numerical invariants of projective type. As an application we study that way geometry of bisectors in $\mathbb{C}H^n$.

References

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