



# UNIVERSIDAD DE GRANADA

Máster Universitario en  
Matemáticas

## Actividad Formativa del máster y del doctorado en matemáticas.

12/11/2018

Actividad Formativa del máster y del doctorado en matemáticas.

Minicurso impartido como actividad formativa del máster y del doctorado en matemáticas.

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Título: COMPLEX HYPERBOLIC GEOMETRY

Abstract: The  $n$ -dimensional complex hyperbolic space  $\mathbb{CH}^n$  consists of classes of negative vectors with respect to Hermitian form of signature  $(n,1)$  in  $\mathbb{C}^{n+1}$ . There are two useful models in  $\mathbb{C}^n$ : the ball model and the paraboloid model which are closely related to the ball and half-space models in the real case. In particular considerations, we restrict to one of the models and observe examples in (complex) dimension 2.



$\mathbb{CH}^n$  with the Hermitian form is an Hadamard manifold of strictly negative sectional curvature ranging from  $-1/4$  (real directions) to  $-1$  (complex directions) so it has visible ideal boundary. In the paraboloid model the ideal boundary has a structure of Heisenberg group.

In  $\mathbb{CH}^n$  there are two types of totally geodesic submanifolds. One of them, a complex hyperbolic subspace, comes from intersection of  $\mathbb{CH}^n$  by a complex linear subspace and is isometric to  $\mathbb{CH}^k$  for  $k < n$ . The others are totally real hyperbolic subspaces coming from totally real linear subspaces. They are embedding of rescaled  $H_k$ 's. On the ideal boundary we observe chains being boundaries of complex hyperbolic subspaces.

In  $\mathbb{CH}^n$  no totally geodesic hypersurface (of real codimension 1) exists although

bisectors (i.e. hypersurfaces equidistant from psir of points) have interesting properties. The isometry group  $\text{PU}(n,1)$  of  $\text{CH}^n$  consists of classes of matrices preserving the Hermitian form. We study them with connection to their boundary action.

As a final, we give distance formulae for geodesic, complex geodesics and hyperspaces. For it, we develop some numerical invariants of projective type. As an application we study that way geometry of bisectors in  $\text{CH}^n$ .

Fecha y lugar: 12, 13 y 14 de noviembre de 2018 de 10:00 a 12:00 horas en el seminario de la primera planta del IEMath-Gr

## Ficheros Adjuntos

- chg\_abs.pdf