Banach limits

Evgeny Semenov
Voronezh, Russia

A linear functional $B \in l_\infty^*$ is called a Banach limit if

1. $B \geq 0$, i. e. $Bx \geq 0$ for $x \geq 0$ and $B1 = 1$.
2. $B(Tx) = B(x)$ for all $x \in l_\infty$, where T is a shift operator, i. e.

$$T(x_1, x_2, \ldots) = (x_2, x_3, \ldots).$$

The existence of Banach limits was proven by S. Banach in his book. It follows from the definition, that $Bx = \lim_{n\to\infty} x_n$ for every convergent sequence $x \in l_\infty$ and $\|B\|_{l^*\infty} = 1$. Denote the set of all Banach limits by $\mathfrak{B}$. It is clear that $\mathfrak{B}$ is a closed convex subset of the unit sphere of the space l_∞^*. Hence, $\|B_1 - B_2\| \leq 2$ for every $B_1, B_2 \in \mathfrak{B}$.

The set $A \subset l_\infty$ is called the set of uniqueness if the fact that two Banach limits B_1 and B_2 coincide on A implies that $B_1 = B_2$.

It was shown that under some restrictions on the operator H, acting on l_∞, there exists such $B \in \mathfrak{B}$ that $Bx = BHx$ for every $x \in l_\infty$. We denote by $\mathfrak{B}(H)$ the set of all such Banach limits.

The sets of uniqueness, invariant Banach limits and extremal points of $\mathfrak{B}$ will be discussed in the talk.

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