Numerical Methods for Ordinary Diffrential Equations.

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March 8, 2019

Abstract

For $I = [t_1, t_1 + T] \subset \mathbb{R}$, let $f(t, \boldsymbol{x}) \in C(I \times \mathbb{R}^m, \mathbb{R}^m)$. With an initial condition $\boldsymbol{\eta} \in \mathbb{R}^m$, we are looking for a function $\boldsymbol{y} \in C^1(I, \mathbb{R}^m)$ solving the **Cauchy problem**

$$\begin{cases} \boldsymbol{y}'(t) = f(t, \boldsymbol{y}(t)), \forall t \in I, \\ \boldsymbol{y}(t_1) = \boldsymbol{\eta}. \end{cases}$$

This is an initial value problem for a system of first order differential equations.
Numerical schemes: We compute approximations $\boldsymbol{u}_i \approx \boldsymbol{y}(t_i)$ on a **partition** $\boldsymbol{t} : t_1 < t_2 < \ldots < t_{n+1} = t_1 + T$ of the interval $I = [t_1, t_1 + T]$. Here $\boldsymbol{u}_1 \in \mathbb{R}^m$ is given and

$$\boldsymbol{u}_{i+1} := \boldsymbol{u}_i + h_i \phi(t_i, \boldsymbol{u}_i, h_i),, \quad h_i := t_{i+1} - t_i, \quad i = 1, \ldots, n. \tag{1}$$

We will define, **stability, order** and **convergence** of the numerical scheme.
Program: If $f \in C\big([t_1, t_1 + T] \times \mathbb{R}^m, \mathbb{R}^m\big)$ and $\boldsymbol{\eta} \in \mathbb{R}^m$, the solution of the Cauchy problem can be approximated by a numerical method using a uniform step length:

```
[t,u]=method(namefunc,I,eta,n)
```
(2)

with inputs the file `namefunc.m` computing the function f, the interval $I = [t_1, t_1 + T]$, the initial condition $\boldsymbol{\eta}$ and the number of intervals in the partition n. The outputs are the partition $\boldsymbol{t}$ and the approximation $\boldsymbol{u} \in \mathbb{R}^{m \times (n+1)}$ of the solution at the grid points.