

On the extension of isometries between the unit spheres of two Banach spaces

Antonio M. Peralta
University of Granada

March, 2017

Abstract

One of the lines explored after the famous Mazur-Ulam theorem is the possibility of extending a surjective isometry between the unit spheres of two Banach spaces as a surjective linear isometry between the space. More concretely, let X and Y be Banach spaces, whose unit spheres are denoted by $S(X)$ and $S(Y)$, respectively. Suppose $T : X \rightarrow Y$ is a surjective real linear isometry. The restriction $T|_{S(X)} : S(X) \rightarrow S(Y)$ defines a surjective isometry. The so-called *Tingley's problem*, asks if every surjective isometry $f : S(X) \rightarrow S(Y)$ arises in this way, or equivalently, if every surjective isometry $f : S(X) \rightarrow S(Y)$ admits an extension to a surjective real linear isometry $T : X \rightarrow Y$. Tingley's problem remains open even for surjective isometries between the unit spheres of a pair of 2 dimensional Banach spaces. We shall revisit in this talk most of the main positive answers to Tingley's problem obtained during the last two or three decades.