Towards the Kadec-Pełczyński-Rosenthal subsequence splitting lemma

Antonio M. Peralta
University of Granada

July, 2016

Abstract

In 1962, M.I. Kadec and A. Pelczyński proved that every bounded sequence (f_n) in $L_p[0,1]$ $(1 \leq p < \infty)$ admits a subsequence (f_{n_k}) which can be written in the form $f_{n_k} = g_k + h_k$, where the h_k's have pairwise disjoint supports and the set $\{g_k : k \in \mathbb{N}\}$ is an equi-integrable or relatively weakly compact subset in $L_p[0,1]$. This result is now called the "subsequence splitting property" or the "Kadec-Pelczyński-Rosenthal dichotomy theorem". This result in commutative measure theory relay on the characterization of relatively weakly compact bounded subsets in the dual of a commutative C*-algebra established by A. Grothendieck.

We shall survey how these result in commutative measure theory have been generalized to what is called "non-commutative measure theory" by the study of the dual of a general C*-algebra. In the non-commutative setting relatively weakly compact subsets were characterized by M. Takesaki and C.A. Akemann, while the Kadec-Pelczyński-Rosenthal dichotomy theorem in the non-commutative setting was independently established by N. Randrianantoanina (2002) and Y. Raynaud and Q. Xu (2003). To illustrate how the non-commutative measure theory is being developed, we shall present new generalizations of these results in the wider setting of JB*-algebras and JB*-triples.